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COMMENT

Hole distribution in Eden clusters

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Abstract. Clusters grown by the Eden process have few empty sites in their interior; these sites are concentrated near the cluster surface. The number of clusters of connected empty sites is investigated by computer simulation and shown to disagree with a simple percolation approximation.

In the usual Eden cluster growth process [1, 2], a cluster grows by occupation of a randomly selected perimeter site at every time step. The perimeter is defined as the set of empty sites which are nearest neighbours of the Eden clusters existing at that time. It is known [3] that far in the interior of the Eden cluster very few empty sites remain. The structure of the surface, on the other hand, is quite complicated [2, 4], and most of the perimeter sites are located there. By definition, we have only one cluster of occupied sites.

Nevertheless, a set of finite clusters can be defined looking instead at the empty sites. These empty sites form an infinite network surrounding the finite Eden cluster of occupied sites. Moreover, these empty sites form finite holes of neighbouring empty sites located in the interior of the Eden cluster. We call the number of such holes N_s with s empty sites.

If the distribution of empty sites, for a fixed distance from the surface, was completely random, then percolation theory [5] could be applied and each site could be assumed to be empty with probability p, with p decreasing with increasing distance from the surface. Then the ratio of the number N_1 of isolated empty sites to the total number N_0 of empty sites at this distance from the surface is

$$N_1/N_0=(1-p)^q$$

in a lattice where each site has q neighbours. Most of these holes will be located near the surface and the hole numbers N_s would decay with s according to a power law.

However, these relations between percolation theory and Eden holes are highly speculative for a variety of reasons.

(i) The main contribution to the hole numbers comes from the region very close to the surface where the assumptions may be invalid, in particular for hole radii larger than the thickness of this surface layer.

(ii) The clustering properties of percolation are influenced if the occupation probability p is not spatially homogeneous [6].

(iii) Empty sites without an occupied neighbour behave different from empty sites with at least one occupied neighbour.

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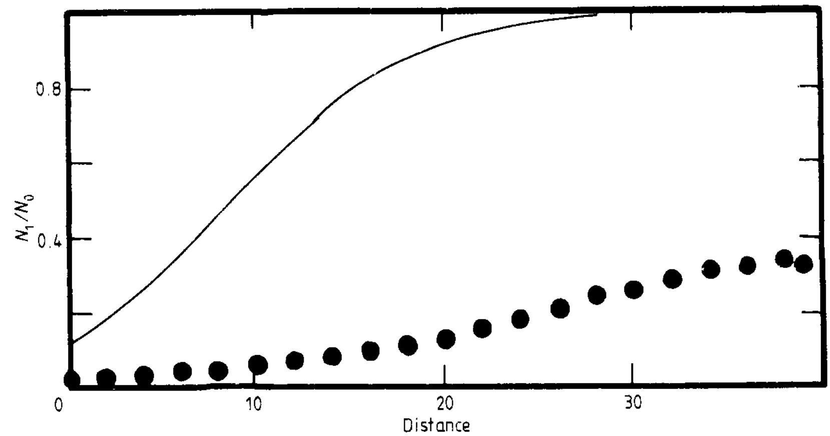

Figure 1. Ratio of number of isolated empty sites to total number of empty sites as a function of distance from the surface. The dots are our Monte Carlo data, the curve the prediction $(1-p)^4$ with our empirically determined concentration p of empty sites.

(iv) The holes near the surface are formed by closing of overhangs, which is not a random process.

(v) The roughening of the surface [4] may dominate over the spatial variation of p.

We have done some computer simulations on the square lattice to test these predictions. To avoid any problems arising from the anisotropy of the Eden process, we worked on strips of width $L = 16$-512, with lengths up to 13×10^6, storing only the growth zone near the surface. Thus clusters containing 1024×10^6 sites were produced on a medium-size CDC Cyber 74 computer; our figures are based on one strip of size 512×10^6, with 13 370 samples of the surface taken during this single growth process.

We analysed the clustering of the empty sites by the Hoshen-Kopelman algorithm [5]. Holes of unit size, i.e. isolated empty empty sites, were also identified directly by looking at their four neighbours and the spatial distribution of these empty sites was determined as a function of their distance from the surface.

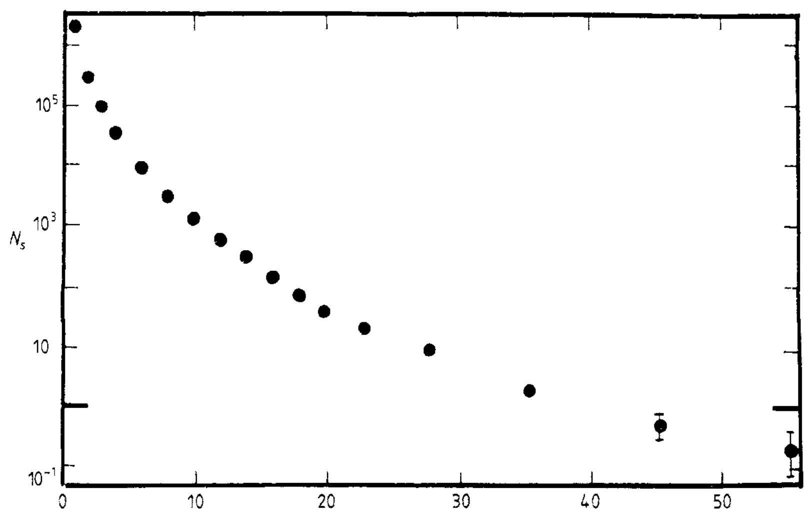

Figure 2. Logarithmic plot of the number of holes with s sites against s.

We found that our percolation approximation is wrong. In particular, the spatial variation of the relative concentration N_1/N_0 does not at all follow $(1-p)^4$ but is much smaller (figure 1). In other words, when only a few empty sites are left, the fraction of sites belonging to holes of size 2, 3, 4, ..., etc, is much larger than that predicted by a random distribution of empty sites. Thus the complicated history of the non-random hole formation at the surface is not forgotten later in the process. For this reason, the usual percolation concepts cannot apply here. The hole numbers seem to decay with increasing hole size, stronger than with a simple power law but weaker than with a simple exponential (figure 2). Instead, a stretched exponential, $\log N_s \propto -s^\sigma$ fits reasonably, with σ near $\frac{1}{4}$ and perhaps increasing with increasing strip width L.

In summary, a percolation description is thorougly wrong and a better theory is needed to explain our data.

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